JOURNAL OF COMPUTATIONAL PHYSICS 98, 108–118 (1992)

A Consistently Formulated QUICK Scheme for Fast and Stable Convergence Using Finite-Volume Iterative Calculation Procedures

T. HAYASE,* J. A. C. HUMPHREY, AND R. GREIF

Department of Mechanical Engineering, University of California at Berkeley, Berkeley, California 94720

Received January 31, 1990; revised November 13, 1990

Previous applications of QUICK for the discretization of convective transport terms in finite-volume calculation procedures have failed to employ a rigorous and systematic approach for consistently deriving this finite difference scheme. Instead, earlier formulations have been established numerically, by trial and error. The new formulation for QUICK presented here is obtained by requiring that it satisfy four rules that guarantee physically realistic numerical solutions having overall balance. Careful testing performed for the wall-driven square enclosure flow configuration shows that the consistently derived version of QUICK is more stable and converges faster than any of the formulations previously employed. This testing includes the relative evaluation of boundary conditions approximated by second- and third-order finite-difference schemes as well as calculations performed at higher Reynolds numbers than previously reported. © 1992 Academic Press, Inc.

1. INTRODUCTION

Soon after Leonard's [1] publication of the QUICK scheme for the discretization of convective transport terms, a series of papers appeared in the literature (Leschziner [2], Han *et al.* [3], Pollard and Siu [4], Freitas *et al.* [5], Perng and Street [6]) addressing the practical implementation and testing of the scheme in flows more complex than those originally inspected by Leonard [1]. Among the configurations closely examined by most of these authors have been the 2D and 3D wall-driven enclosure flows. Certainly, in the 2D case, this configuration has become a standard for the testing of numerical calculation procedures (see Ghia *et al.* [7], Schreiber and Keller [8], and Choi *et al.* [9]) even though it represents the idealization of a flow whose practical realization seems improbable (see Koseff and Street [10] and the discussion therein by Humphrey).

In spite of its impracticability the 2D wall-driven enclosure flow represents an excellent test case for evaluating convective differencing schemes. This is because of the large streamline-to-grid skewness present over most of the flow (on a rectangular grid) and the existence of several relatively large, recirculation regions where diffusion and convection transport terms are of comparable magnitude, thus requiring a finite-difference representation of the latter at least as accurate as for the former.

The QUICK scheme employs a three-point upstream-weighted quadratic interpolation technique within the context of a control-volume approach for calculating on a staggered grid. Leonard [1] has shown that this procedure has greater formal accuracy than the central difference scheme and retains the basic stable convective sensitivity property that is characteristic of upstream-weighted schemes. Figure 1 provides the basis for a 1D control volume formulation for the transport of the scalar quantity ϕ. A uniform grid is shown but neither this simplification nor the 1D transport assumption precludes the application of the findings of our investigation to 2D and 3D flows on nonuniform grids. By reference to Fig. 1, control volume surface values for ϕ are obtained by fitting a parabola to the values of ϕ at three consecutive nodes: the two nodes located on either side of the surface of interest, plus the adjacent node on the upstream side. In this way, Leonard [1] finds for $u_e > 0$, $u_w > 0$,

$$\begin{aligned}\phi_e &= 1/2(\phi_i + \phi_{i+1}) - 1/8(\phi_{i-1} - 2\phi_i + \phi_{i+1})\\ \phi_w &= 1/2(\phi_{i-1} + \phi_i) - 1/8(\phi_{i-2} - 2\phi_{i-1} + \phi_i).\end{aligned} \tag{1}$$

Similar expressions can be found when $u_e < 0$ and $u_w < 0$, and they can all be interpreted as linear interpolations for ϕ_e and ϕ_w corrected by the inclusion of terms proportional to the respective upstream-weighted curvatures.

As in the numerical studies mentioned above, we wish to use QUICK to calculate accurate values of the dependent variable ϕ at the surfaces of a control volume while avoiding

* Department of Mechanical Engineering, Nagoya University, Nagoya, 464-01 Japan.

0021-9991/92 $3.00

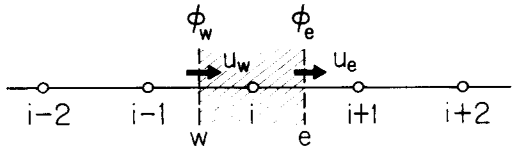


FIG. 1. One-dimensional, uniform, staggered grid showing the nodes involved in the evaluation of ϕ at the east (e) and west (w) surfaces of a control volume or cell centered at node i.

high convection rate instabilities. We would, furthermore, like to do this with a consistently derived version of QUICK. This is achieved by subjecting the final determination of the weighting coefficients in the scheme to the observance of some specific rules, as opposed to the rather ad hoc numerical explorations and tests that have been performed in the past to determine these coefficients.

2. OPTIMIZATION OF THE QUICK SCHEME FORMULATION

2.1. Generalization of QUICK

All QUICK scheme expressions communicated in the literature derive from Leonard's original form given by Eq. (1). In these expressions some terms are evaluated using previously calculated values in the iteration process and are referred to as "source terms." A generalization of Eq. (1) can be postulated, by stating that:

$$\begin{aligned} &u_e > 0,\ u_w > 0, \\ &\qquad \phi_e = a_1\phi_{i-1} + a_2\phi_i + a_3\phi_{i+1} + S_e^+ \\ &\qquad \phi_w = b_1\phi_{i-1} + b_2\phi_i + b_3\phi_{i+1} + S_w^+; \\ &u_e < 0,\ u_w < 0, \\ &\qquad \phi_e = b_3\phi_{i-1} + b_2\phi_i + b_1\phi_{i+1} + S_e^- \\ &\qquad \phi_w = a_3\phi_{i-1} + a_2\phi_i + a_1\phi_{i+1} + S_w^-; \end{aligned} \tag{2}$$

where S_e^+, S_w^+, S_e^-, and S_w^- are source terms written as

$$\begin{aligned} S_e^+ &= (-\tfrac{1}{8} - a_1)\,\phi_{i-1} + (\tfrac{3}{4} - a_2)\,\phi_i + (\tfrac{3}{8} - a_3)\,\phi_{i+1} \\ S_w^+ &= -\tfrac{1}{8}\phi_{i-2} + (\tfrac{3}{4} - b_1)\,\phi_{i-1} + (\tfrac{3}{8} - b_2)\,\phi_i - b_3\phi_{i+1} \\ S_e^- &= -\tfrac{1}{8}\phi_{i+2} + (3/4 - b_1)\,\phi_{i+1} + (\tfrac{3}{8} - b_2)\,\phi_i - b_3\phi_{i-1} \\ S_w^- &= (-\tfrac{1}{8} - a_1)\,\phi_{i+1} + (\tfrac{3}{4} - a_2)\,\phi_i + (\tfrac{3}{8} - a_3)\,\phi_{i-1}. \end{aligned} \tag{3}$$

The "a" and "b" column entries in Table I show the values determined and used for these coefficients in earlier studies. By contrast, the last row consisting of 0's and 1's, corresponds to the values obtained in this study. Their determination is the subject of the next section.

2.2. Consistent Determination of the QUICK Scheme Coefficients

In the absence of any source terms and using central differencing for the diffusion term, the one-dimensional, steady-state, convective-diffusion difference equation for the quantity ϕ obtained via a finite-volume approach is readily shown to be

$$\begin{aligned} (F_e^+ + F_e^-)\,\phi_e - (F_w^+ + F_w^-)\,\phi_w \\ = D_e(\phi_{i+1} - \phi_i) - D_w(\phi_i - \phi_{i-1}), \end{aligned} \tag{4}$$

where

$$F_e^+ = \begin{cases} \rho u_e & (u_e > 0) \\ 0 & (u_e \leqslant 0) \end{cases}, \qquad F_e^- = \begin{cases} 0 & (u_e > 0) \\ \rho u_e & (u_e \leqslant 0) \end{cases}, \qquad \text{etc.}$$

$$D_e = \Gamma_e/\Delta x_e, \qquad D_w = \Gamma_w/\Delta x_w. \tag{5}$$

Substitution of Eqs. (2) and (3) into Eq. (4) and rearranging terms gives an equation for the quantity ϕ_i in terms of its neighbors,

$$B_P\phi_i = B_E\phi_{i+1} + B_W\phi_{i-1} + S, \tag{6}$$

TABLE I

Values of "a" and "b" Coefficients Used in Various QUICK Schemes

	Coefficients						Rules				
	a_1	a_2	a_3	b_1	b_2	b_3	R_1	R_2	R_3	R_4	R_5
Leschziner (1980)	$-\frac{1}{8}$	$\frac{6}{8}$	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{3}{8}$	0	P	P	Y	N	P
Han *et al.* (1981)	0	$\frac{6}{8}$	$\frac{4}{8}$	$\frac{4}{8}$	$\frac{3}{8}$	0	N	P	Y	N	P
Pollard and Siu (1982)	$-\frac{1}{8}$	$\frac{7}{8}$	$-\frac{6}{8}$	$\frac{6}{8}$	$-\frac{6}{8}$	0	N	Y	Y	Y	N
Freitas *et al.* (1985)	$-\frac{1}{8}$	$\frac{6}{8}$	$-\frac{1}{8}$	$\frac{6}{8}$	$\frac{3}{8}$	0	N	Y	Y	N	N
Present work	0	1	0	1	0	0	Y	Y	Y	Y	Y

Note. Level of rule satisfaction: Y (yes); N (no); P (partly).

where

$$\begin{aligned} B_P &= a_2 F_e^+ + b_2 F_e^- - b_2 F_w^+ - a_2 F_w^- + (D_e + D_w) \\ B_E &= -a_3 F_e^+ - b_1 F_e^- + b_3 F_w^+ + a_1 F_w^- + D_e \\ B_W &= -a_1 F_e^+ - b_3 F_e^- + b_1 F_w^+ + a_3 F_w^- + D_w \\ S &= -S_e^+ F_e^+ - S_e^- F_e^- + S_w^+ F_w^+ + S_w^- F_w^-. \end{aligned} \tag{7}$$

Note that B_P can also be written

$$B_P = B_E + B_W + B_O, \tag{8}$$

where

$$\begin{aligned} B_O = {} & (a_1 + a_2 + a_3) F_e^+ + (b_1 + b_2 + b_3) F_e^- \\ & - (b_1 + b_2 + b_3) F_w^+ - (a_1 + a_2 + a_3) F_w^-. \end{aligned}$$

The Rules for Evaluating the "a" and "b" Coefficients

The solution of the finite difference equations corresponding to Eq. (6) in this and the studies by Han *et al.* [3], Pollard and Siu [4] and Leschziner [2] are based on the SIMPLE algorithm (or a variation of this algorithm, like SIMPLER) documented by Pantankar [11]. For a 2D flow, the x- and y-velocity components are solved line-by-line assuming a fixed pressure field. After sweeping the entire solution domain the pressure field is adjusted to ensure the satisfaction of continuity along every line of cells. This destroys the compliance of the velocity and pressure fields with the momentum equations. Thus, it is necessary to iterature upon the momentum and continuity equations until they are simultaneously satisfied to the accuracy required.

Four rules, given by Patankar [11], ensure the stable convergence of a finite-volume-based algorithm towards a physically realistic numerical solution. These are:

RULE 1. Consistency of surface flux calculations at the control volume faces.

This rule requires that the value of the flux of a quantity ϕ across a control volume surface be independent of the side of the surface from which the flux is evaluated. Thus, the flux that leaves a given control volume through a particular surface must be identical to the flux that enters the adjacent control volume sharing that surface.

Assuming incompressible one-dimensional flow, this rule requires that:

$$(\phi_e)_i = (\phi_w)_{i+1}, \tag{9}$$

where i and $i+1$ refer to the nodes with respect to which ϕ_e and ϕ_w are respectively evaluated. Applying Eq. (9) to Eqs. (2) and (3) yields

$$a_1 = 0, \qquad a_2 = b_1, \qquad a_3 = b_2, \qquad 0 = b_3. \tag{10}$$

Column "R_1" in Table I shows that all previous QUICK scheme formulations failed to rigorously satisfy this constraint.

RULE 2. All coefficients positive in the discretization equation.

As illustrated in Eq. (6), the balance of ϕ at a grid point i (ϕ_i) is influenced by the values of ϕ at the adjacent locations (ϕ_{i+1} and ϕ_{i-1}). When the transport of ϕ is by convection and diffusion alone, an increase in the value of ϕ at one grid point should lead to an increase in the value of ϕ at a neighboring grid point. Thus, an increase in ϕ_{i-1} (or ϕ_{i+1}) must lead to an increase in ϕ_i and, therefore, it follows that the coefficients B_E, B_W, and B_P must all have the same sign; in the present case chosen to be positive:

$$B_E \geqslant 0, \qquad B_W \geqslant 0, \qquad B_P \geqslant 0. \tag{11}$$

Applying the inequality expressed in Eq. (11) to the expression given by Eq. (7) yields

$$a_1 \leqslant b_1, \qquad a_2 \geqslant b_2, \qquad a_3 \leqslant b_3. \tag{12}$$

The source terms in iterative solution procedures are evaluated using values from the former iteration step. In analogy with the time dependent calculation, the positive coefficient condition should also apply to coefficients in the source terms. Unfortunately, it can be shown that there do not exist a set of coefficients which simultaneously satisfy expressions (12) and the additional constraint for the source terms.

RULE 3. Negative-slope linearization of the source term.

In the presence of sources a more general form of Eq. (8) is (Pantankar [11])

$$B_P = B_E + B_W + B_O - S_P. \tag{13}$$

To avoid the possibility that $B_P < 0$ due to a large positive S_P (and, as a result, violate Rule 2), Rule 3 requires that when a source term is linearized according to $S = S_C + S_P \phi_i$, the coefficient S_P should be $S_P \leqslant 0$. Because the linearization of the source term, S, does not depend on the particular differencing schemes used for convection or diffusion, Rule 3 can be satisfied for all schemes and, therefore, plays no immediate role in determining the coefficients of interest.

RULE 4. Node coefficient equal to the sum of neighboring node coefficients.

This rule stipulates that

$$B_P = B_E + B_W \tag{14}$$

and is a consequence of requiring that both ϕ and $\phi + C$, where C is a constant, should satisfy the finite difference approximation of the differential transport equation. The rule can be strictly enforced only when derivatives of ϕ alone appear in the original transport equation. For example, the presence of a source term that depends on ϕ disallows the rule. Notwithstanding, requiring that Eq. (14) be satisfied by a differencing scheme is important for generating physically meaningful results. Applying Eq. (14) to Eq. (8) yields

$$a_1 + a_2 + a_3 = b_1 + b_2 + b_3. \tag{15}$$

RULE 5. Constraints on the summations of coefficients.

Rules 1–4 translate into the constraints given by Eqs. (10), (12), and (15) for the coefficients of a generalized QUICK formulation. Combining these relations yields

$$\begin{aligned} a_1 &= 0 \\ a_2 &= b_1 \geqslant 0 \\ a_3 &= b_2 \leqslant 0 \\ b_3 &= 0. \end{aligned} \tag{16}$$

We now impose an additional requirement for the QUICK representation. This is that ϕ_e and ϕ_w should be expressible in terms of weighted values of the neighboring ϕ's and the remaining source terms according to

$$\begin{aligned} \sum_{j=1}^{3} a_j &= 1, \qquad a_j \geqslant 0 \\ \sum_{j=1}^{3} b_j &= 1, \qquad b_j \geqslant 0. \end{aligned} \tag{17}$$

Together with Eqs. (16) these additional constraints yield an unambiguous determination of the coefficients as follows:

$$\begin{aligned} a_1 &= 0 \\ a_2 &= b_1 = 1 \\ a_3 &= b_2 = 0 \\ b_3 &= 0. \end{aligned} \tag{18}$$

Thus, the corresponding expressions for Eqs. (2) are

$u_e > 0,\ u_w > 0,$

$$\begin{aligned} \phi_e &= \phi_i + \tfrac{1}{8}(-\phi_{i-1} - 2\phi_i + 3\phi_{i+1}) \\ \phi_w &= \phi_{i-1} + \tfrac{1}{8}(-\phi_{i-2} - 2\phi_{i-1} + 3\phi_i); \end{aligned} \tag{19a}$$

$u_e < 0,\ u_w < 0,$

$$\begin{aligned} \phi_e &= \phi_{i+1} + \tfrac{1}{8}(3\phi_i - 2\phi_{i+1} - \phi_{i+2}) \\ \phi_w &= \phi_i + \tfrac{1}{8}(3\phi_{i-1} - 2\phi_i - \phi_{i+1}); \end{aligned} \tag{19b}$$

where the source terms are in parentheses.

This form of the QUICK formulation has a very simple structure; the value of ϕ is given as the sum of an upwind estimation with correction source terms. In addition to listing the "a" and "b" coefficients used in earlier works, Table I shows the compliance of the various QUICK schemes proposed with respect to the rules constraining these coefficients. Of all the schemes, only the present QUICK formulation satisfies all five constraints. However, this does not mean that the present formulation is the only possible choice of a QUICK scheme. It may be possible to find other equally accurate formulations which satisfy the rules invoked here but which do not comply with the general form given by Eq. (2). For example, Thompson and Ferziger [12] have mentioned a possible "defect-correction procedure," where QUICK is written as the sum of the first-order power law method and remaining source terms. The present QUICK formulation, on the other hand, yields a defect-correction scheme based on the upwind difference approximation. Among the possible formulations, however, the present QUICK scheme formulation yields the simplest algebraic form.

3. TEST CALCULATIONS FOR WALL-DRIVEN SQUARE ENCLOSURE FLOW

Numerical calculations were performed for the 2D wall-driven square enclosure flow configuration previously investigated by Han *et al.* [3], Pollard and Siu [4] and Freitas *et al.* [5] using their respective QUICK schemes and the formulation proposed here. The calculations were conducted as explained by Han *et al.* except for the following: (a) The SIMPLER algorithm was used in place of SIMPLE (see Patankar [11]) to calculate pressure and update velocities; (b) The MSI method of Schneider and Zedan [13] was used in place of the Thomas algorithm to solve the algebraic system of finite difference equations; (c) Calculations were performed assuming steady state flow and the false transient approach of Han *et al.* was not used to stabilize the computations. However, converged solutions could not be obtained for $\mathrm{Re} = 10^4$ on grids finer than 40×40 nodes.[1] To achieve stable, converged solutions on fine grids at high Re, it was necessary to retain the

[1] Pollard and Siu [4] obtained a steady solution of the wall-driven enclosure flow for $\mathrm{Re} = 10^5$ on a coarse 10×10 grid. A steady solution was also obtained with the present QUICK formulation for the same conditions, but cannot be expected to be accurate due to the extremely coarse resolution of the grid.

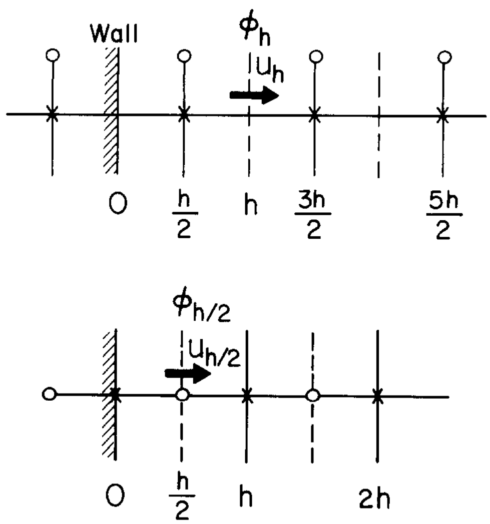


FIG. 2. Second- and third-order representations near the enclosure boundaries: ○, main (scalar) grid point; ×, staggered (velocity) grid point. First: ϕ is parallel to wall; evaluation of ϕ_h; $u_h<0$, $\phi_{(3/2)h}+\frac{1}{8}(3\phi_{h/2}-2\phi_{(3/2)h}-\phi_{(5/2)h})$ (QUICK); $u_h>0$, $\phi_{h/2}+\frac{1}{2}(\phi_{(3/2)h}-\phi_{h/2})$ (2nd order), $\phi_{h/2}+\frac{1}{3}(\phi_{(3/2)h}-\phi_0)$ (3rd order). Second: ϕ is perpendicular to wall; evaluation of $\phi_{h/2}$; $u_{h/2}<0$, $\phi_h+\frac{1}{8}(3\phi_0-2\phi_h-\phi_{2h})$ (QUICK); $u_{h/2}>0$, $\phi_0+\frac{1}{2}(\phi_h-\phi_0)$ (2nd order), $\phi_0+\frac{1}{8}(-5\phi_0+6\phi_h-\phi_{2h})$ (3rd order). Source terms are in parentheses.

unsteady terms in the momentum equations. These terms were approximated using an Euler implicit difference scheme. Constant property laminar flows were assumed for the Reynolds numbers investigated ranging from 10^2 to 10^4. The Reynolds number is defined by $Re = LU/\nu$, where L is the length of the square enclosure side wall, U is the speed of the sliding wall, and ν is the fluid kinematic viscosity. In the following discussion, all variables are non-dimensionalized using L, U, and ρ, the fluid density.

The grids used were evenly spaced and consisted of 10×10, 20×20, 40×40, or 80×80 nodes. Each calculation was terminated when the residual ε became smaller than $\varepsilon_0 = 1\times 10^{-5}$, where ε is defined as the maximum value of the residuals for the mass, u-momentum and v-momentum conservation equations.

The general form of the QUICK scheme formulation may not apply for cells adjacent to walls depending on the flow direction. This is because the scheme may require a value outside the calculation domain (see Fig. 1). The modification of the QUICK scheme near boundary cells is given by Leonard [14]. However, because a third-order boundary treatment[2] sometimes causes instability [5], the second-order boundary treatment persists in QUICK-based solution procedures [6]. In the present work, Leonard's third-order scheme is rewritten as the sum of the upwind evaluation and the remaining source terms, in the same way as the QUICK formulation. Figure 2 illustrates second- and third-order formulations which apply at the boundary cells. A comparison between the second- and third-order boundary representations was performed and is discussed further below.

[2] In this paper, the order of each discretization scheme corresponds to the order of the leading error term in the Taylor series; therefore, the QUICK, central, and upwind difference schemes are third-, second-, and first-order accurate, respectively.

3.1. Comparison Among QUICK, CENTRAL, and HYBRID Schemes

Higher order finite difference schemes generally are less stable than lower order schemes and, therefore, can require more computational effort to generate acceptable numerical solutions. Comparisons of different QUICK scheme formulations with the second-order central difference scheme (CENTRAL) and the first-order upwind difference scheme (when combined with the CENTRAL scheme this is referred to as the HYBRID scheme) have been performed by Han *et al.* and Pollard and Siu for the wall-driven enclosure flow, and by Leschziner for other recirculating flows. However, all of these authors employed second-order accurate finite-difference boundary conditions and their calculations were limited to $Re \leqslant 1000$ by the instability characteristics of their numerically encoded QUICK formulations.

In this section, the QUICK scheme formulation with third-order boundary condition treatment is compared with the CENTRAL and HYBRID schemes for Reynolds numbers up to 10^4. In addition, the effect of using a second-order representation for the boundary cells is discussed for the QUICK scheme.

Because central differencing is used to represent the diffusion transport terms in all the schemes, the labels "QUICK," "CENTRAL," and "HYBRID" (throughout the entire paper) refer to the convective differencing practices.

It is known that central difference approximations for the convection transport terms result in inappropriate difference equations for convection dominated flows. To avoid this problem, the central difference scheme employed here is formulated in the same way as the QUICK scheme; see the Appendix.

The Effect of Reynolds Number

Centerline velocity profiles for the u (x-direction) and v (y-direction) velocity components in a wall-driven square enclosure are shown in Fig. 3a–c for $Re = 10^2$, 10^3, and 10^4. Each figure provides a coarse (20×20) grid comparison among the three differencing schemes. Also shown are the results obtained with QUICK on an (80×80) grid and the results calculated by Ghia *et al.* [7] on finer (129×129 or 257×257) grids as the standards for comparison. The profiles plotted in Fig. 3a, for $Re = 10^2$, show that on a 20×20 grid all three schemes closely approach the fine grid

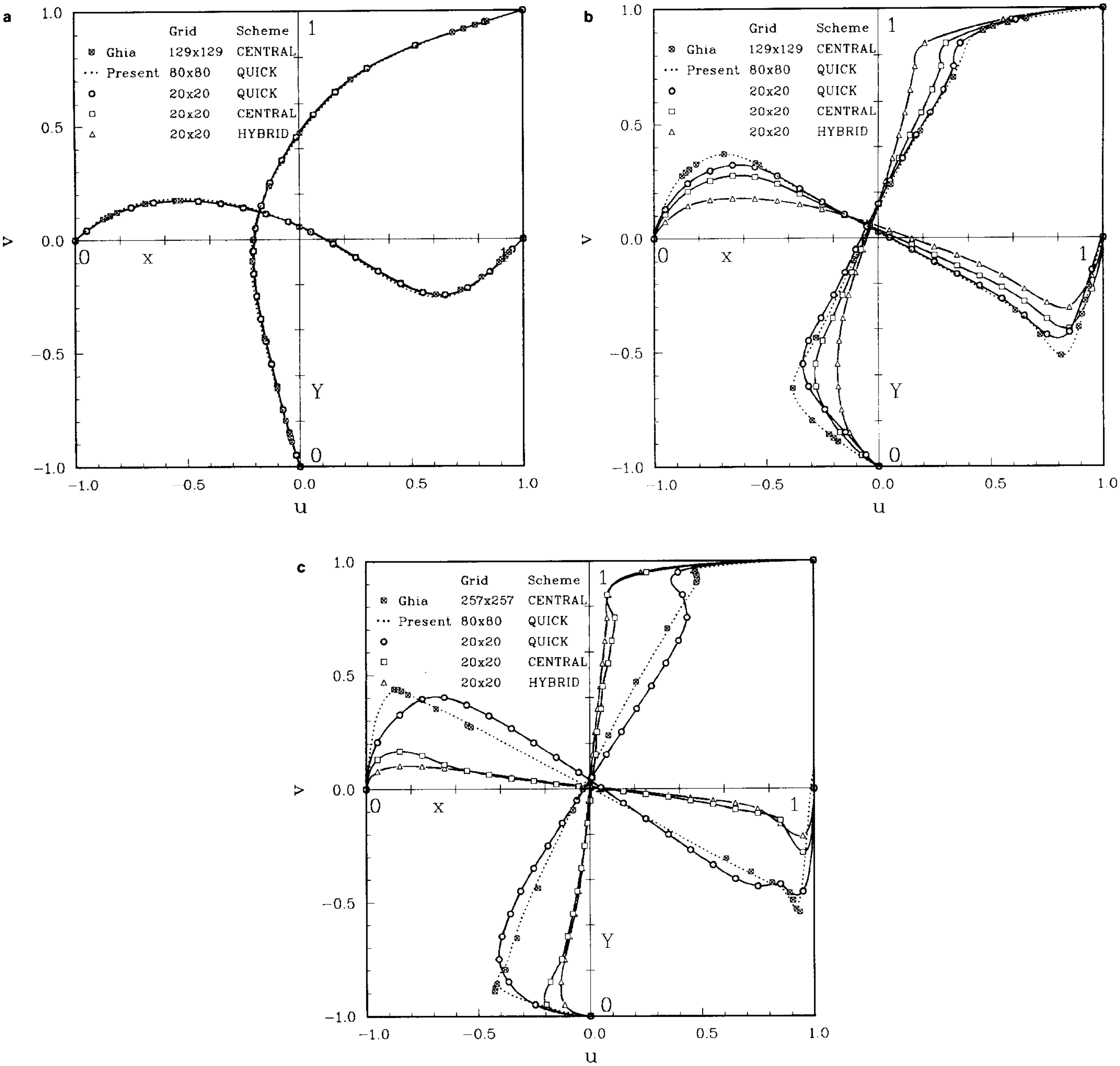


FIG. 3. Non-dimensional horizontal (u) and vertical (v) velocity component profiles along the vertical (y) and horizontal (x) centerlines of a wall-driven square enclosure flow. (a) $Re = 10^2$; (b) $Re = 10^3$; (c) $Re = 10^4$.

solution. However, for $Re = 10^3$ the coarse grid profiles already reveal differences among the schemes and at $Re = 10^4$ the differences are quite significant, especially with respect to the fine grid results.

The extent of the disagreement among schemes appears even more clearly in the streamline patterns compared in Figs. 4a–d. Both the HYBRID and CENTRAL difference scheme calculations on a 20×20 grid fail to reproduce the full *qualitative* detail revealed by the 80×80 QUICK scheme calculations; recirculation regions have been lost and the predicted strength of the primary vortex is significantly less. However, in spite of its imperfections, the 20×20 QUICK scheme solution reproduces surprisingly faithfully all the qualitative features appearing on the finer 80×80 grid.

The Effect of Grid Refinement

An exploration of the influence of grid refinement was performed for the case of $Re = 10^3$ in the wall-driven

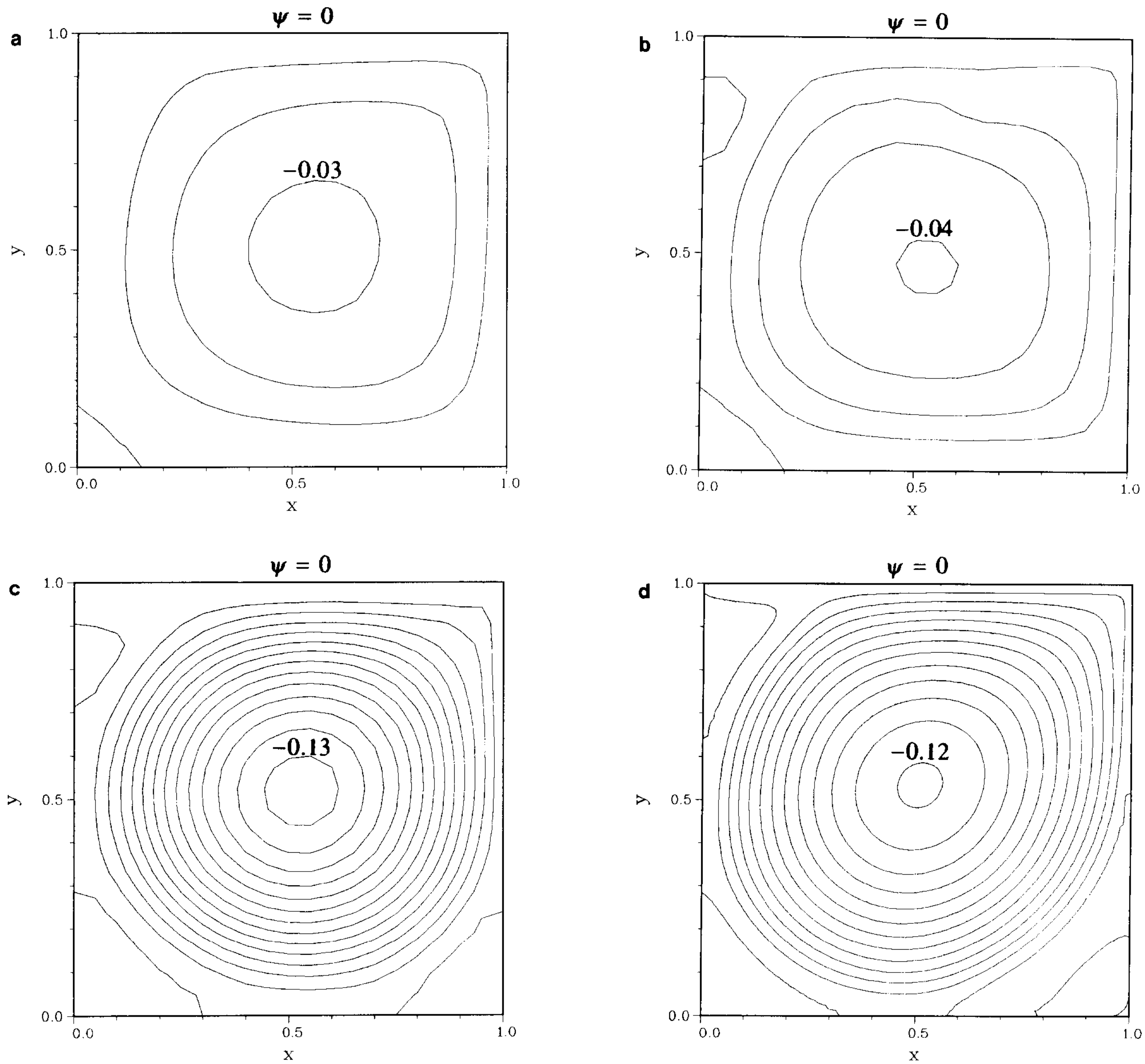


FIG. 4. Calculated streamlines for the wall-driven square enclosure flow with $Re = 10^4$; comparison of three schemes ((a) HYBRID; (b) CENTRAL; (c) QUICK) on a coarse (20×20) grid with QUICK (d) on an 80×80 grid.

enclosure flow. Figure 5 shows the value of the stream function, ψ_c, at the center of the primary vortex plotted as a function of the grid refinement, m, in each coordinate direction. The value of ψ_c is commonly used as a sensitivity measure of the accuracy of solutions. For a given grid refinement, these results quantify the extent to which a higher order scheme outperforms a lower order one. For example, a 20×20 grid with QUICK is equivalent to a 40×40 grid with CENTRAL and an 80×80 grid with HYBRID.

The dotted line in Fig. 5 corresponds to the QUICK scheme using a second-order boundary condition treatment (see Fig. 2). The reduced accuracy at the boundary significantly reduces the overall accuracy of the solution. A further example of the solution degradation as a result of low-order boundary treatment is provided in Fig. 6. This compares centerline velocity profiles on a 40×40 grid with $Re = 10^4$ and shows that the superiority of QUICK over CENTRAL is quickly degraded when a second-order accurate boundary treatment is employed.

3.2. Relative Evaluation of QUICK Scheme Formulations

Having ascertained that the present formulation of the QUICK scheme yields correct and more accurate solutions for the wall-driven enclosure flow than lower order schemes,

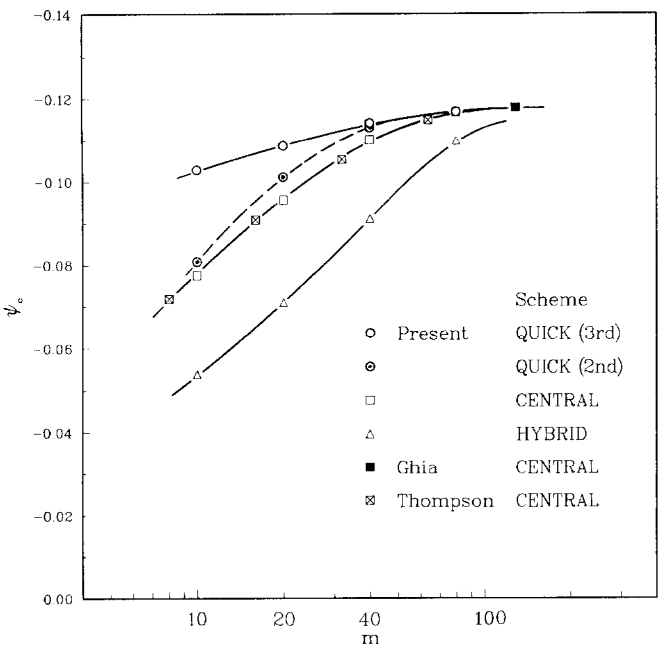


FIG. 5. The value of the stream function at the center of the primary vortex, ψ_c, as a function of the grid refinement, m, in each coordinate direction; $Re = 10^3$ in the wall-driven enclosure flow.

we have proceeded with its evaluation relative to the earlier formulations by Leschziner [2], Han *et al.* [3], Pollard and Siu [4], and Freitas *et al.* [5]. Since all of these formulations derive from Leonard's formulation (see Eq. (1)), their converged solutions are identical. As a result, possible differences among the formulations will be due to their respective stability characteristics.

In general, a stable procedure allows the use of a large under-relaxation factor at each iteration step, resulting in fast convergence to the solution. In this section the performance of the various QUICK scheme formulations was systematically examined for the wall-driven enclosure flow on 10×10, 20×20, and 40×40 grids for Reynolds number ranging from 10^2 to 10^4. Every calculation based on the SIMPLER method with the third-order boundary treatment was performed using zero initial conditions for all the dependent variables (u, v, p). Figure 7 shows the calculated rates of change of the residual ε on a 20×20 grid with $Re = 10^3$. These plots represent the best performances obtained with the corresponding optimum underrelaxation factors, α_{opt} (shown between parentheses), set for each QUICK scheme formulation. The plots show that the present version of QUICK outperforms all previous formulations. In particular, the version of QUICK proposed by Pollard and Siu [4] yields extremely slow reductions of the residuals with increasing iteration number.

Values of the optimum under-relaxation factor were determined through the type of calculations shown in Fig. 8. For each QUICK scheme formulation, the number of iterations required to achieve convergence on a given grid

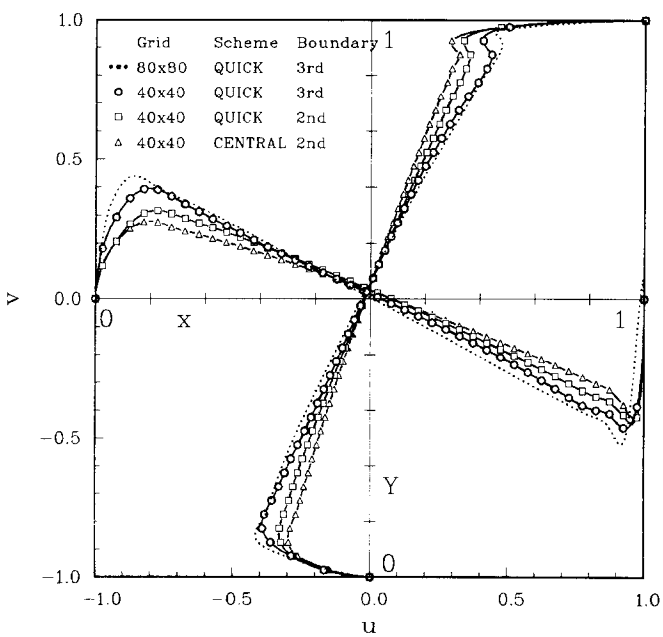


FIG. 6. Non-dimensional u- and v-velocity profiles along the vertical and horizontal centerlines of a wall-driven square enclosure flow with $Re = 10^4$; relative comparison among second- and third-order boundary treatments using the QUICK scheme and the CENTRAL scheme on 40×40 grids.

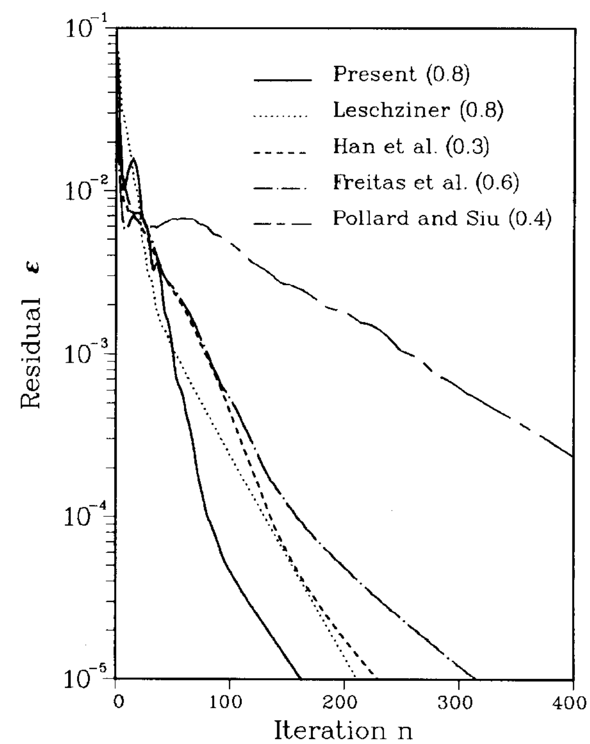


FIG. 7. Variation of the residual, ε, with iteration number, n for the QUICK scheme formulations compared. The optimum value of the under-relaxation factor, α_{opt}, was used for each scheme and is given in parentheses. All calculations were performed on a 20×20 grid for a wall-driven square enclosure flow with $Re = 10^3$.

was plotted as a function of the under-relaxation factor, α. The optimum under-relaxation factor, α_{opt}, is defined as the value of α which corresponds to the minimum number of iterations. The figure shows the superiority of the present scheme and that of Leschziner over the remainder which are much more sensitive to variations of the under-relaxation factor. The value of α_{opt} was determined numerically and its variation with Re on a 20×20 grid is plotted in Fig. 9a for each QUICK scheme formulation. The figure shows that for $Re > 10^2$ the under-relaxation factors for the non-optimized QUICK schemes must be substantially reduced as Re is increased in order to maintain calculation stability. By contrast, the present version of QUICK allows high values of α_{opt} over the whole range of Re explored. The corresponding number of iterations required to achieve a converged solution is plotted as a function of the Reynolds number in Fig. 9b. The result again shows the much better performance of the present formulation.

The next comparison was made among the QUICK scheme formulations with the Reynolds number fixed ($Re = 10^3$) using three different grids of 10×10, 20×20, and 40×40 nodes. Figure 10a shows that the present QUICK scheme allows high values of α_{opt} and that this is relatively insensitive to the grid size.

The variation of iteration number required to achieve convergence as a function of grid refinement is shown in

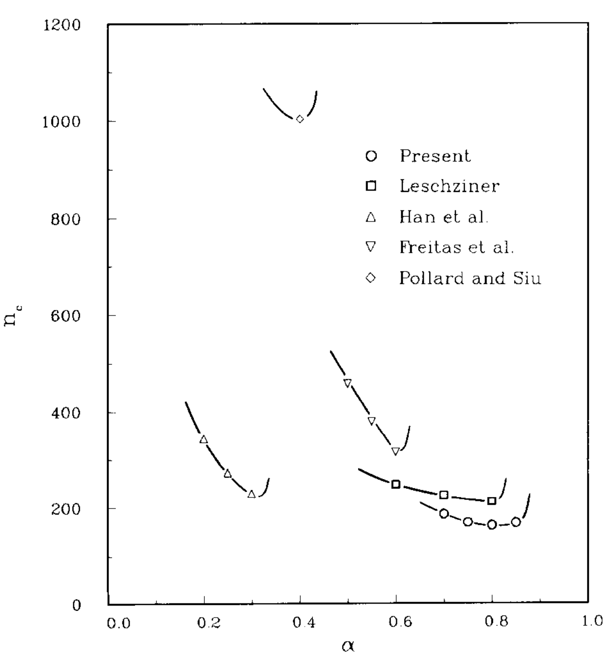


FIG. 8. Number of iterations, n_c, required to achieve convergence ($\varepsilon < 1 \times 10^{-5}$) as a function of the under-relaxation factor α; comparison among QUICK formulations on a 20×20 grid for a wall-driven square enclosure flow with $Re = 10^3$. In the plot, symbols denote converged calculations while the right-hand ends of the curves plotted through the symbols denote unconverged calculations.

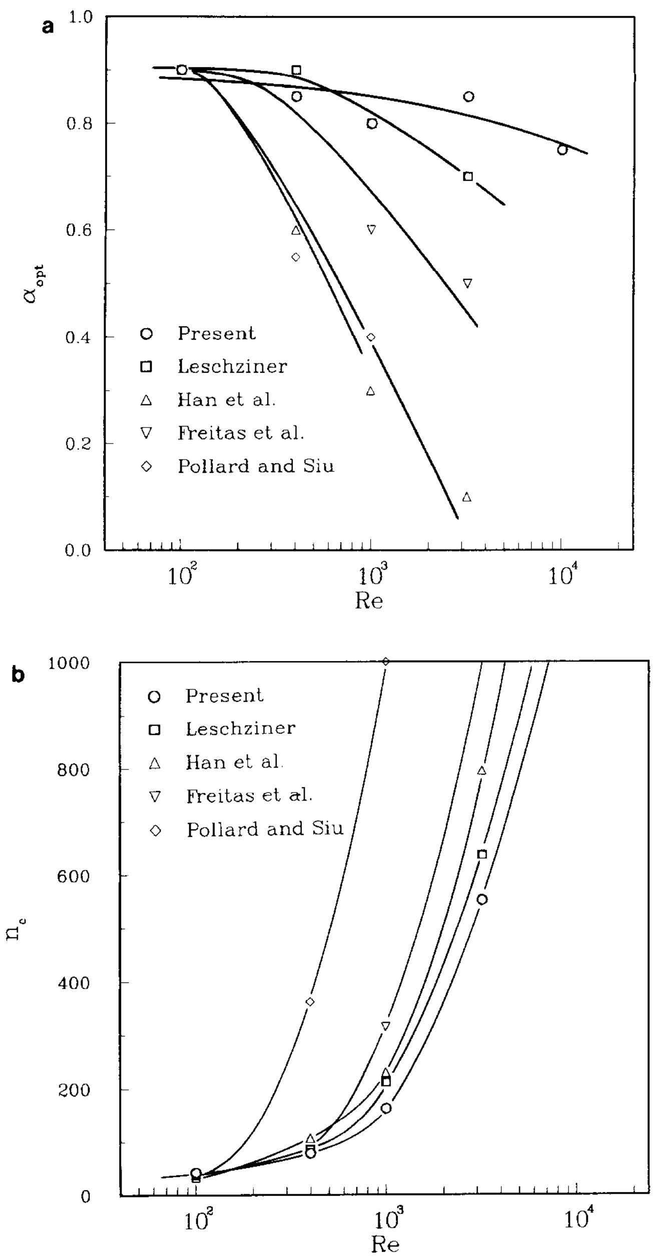
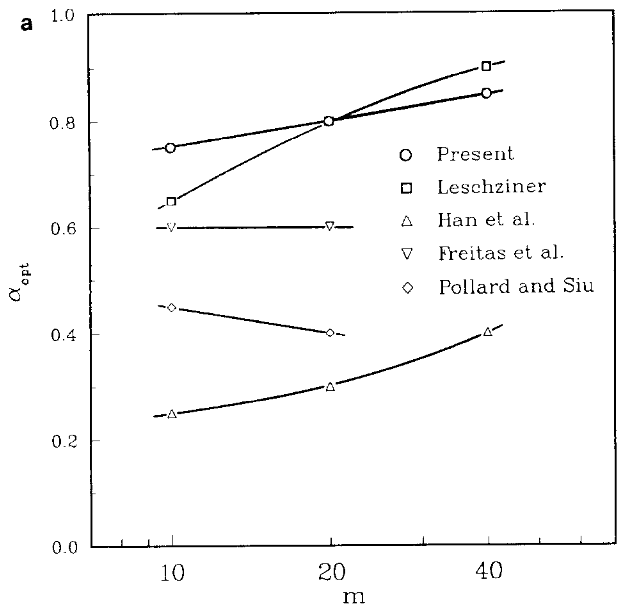


FIG. 9. Effect of Reynolds number on the performance of each QUICK scheme formulation for a wall-driven square enclosure flow on a 20×20 grid: (a) variation of the optimum under-relaxation factor, α_{opt}, with Re. (b) variation of iteration number to convergence, n_c, with Re.

Fig. 10b. This plot also reveals the contrast between the present QUICK scheme and Leschziner's version with respect to the other schemes. The number of iterations for the former is independent of the grid refinement, while it increases monotonically with grid refinement for the latter. The above results reveal the predominant stability and robustness, i.e., insensitivity to parameter variation, of the solution procedure with the present QUICK scheme formulation.



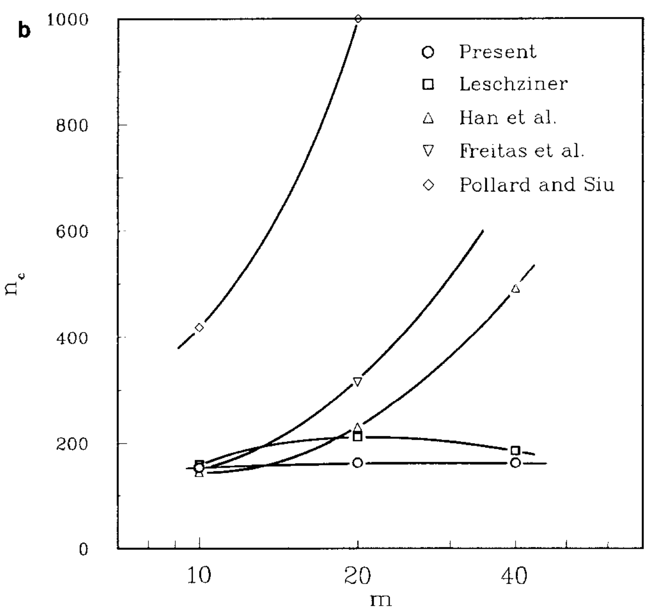


FIG. 10. Effect of the grid refinement, m, in one direction on the performance of each QUICK scheme formulation for the wall-driven square enclosure flow with $\mathrm{Re} = 10^3$; (a) variation of the optimum under-relaxation factor, α_{opt}, with grid refinement, m; (b) variation of iteration number to convergence, n_c, with grid refinement, m.

CONCLUSION

This study places on firm ground a method for consistently deriving an improved formulation of the QUICK scheme which was previously lacking. Consistency is achieved by reference to five rules that guarantee physically realistic and stable numerical solutions of finite volume-approximated conservation equations for mass and momentum. Although not exhaustive, extensive testing of the new QUICK scheme in a complex elliptic flow with strong streamline-to-grid skewness shows that the new formulation is much more robust and converges considerably faster than all previous formulations.

The comparison among schemes performed in this study reveals the superiority of QUICK when employed with a third-order accurate boundary condition treatment, especially at high Reynolds number. Earlier testing performed by Han *et al.* [3], using a formulation of QUICK with a weighting of coefficients that does not fully satisfy the set of rules proposed here for fast and stable convergence, shows that the scheme accurately resolves the viscous dominated region in a stagnation point flow. Notwithstanding, the boundary condition treatment developed in this work represents an improvement over that used by Han *et al.*, since it retains the third-order accuracy of the scheme all the way to the wall.

In concluding, we note that the methodology outlined here, using Rules 1 to 5 in the text to derive a consistent version of the QUICK scheme, can also be applied to derive corresponding consistent versions of discretization schemes of any order.

APPENDIX: FORMULATION OF CENTRAL DIFFERENCE SCHEME

In this scheme, control volume surface values for ϕ are obtained as the mean of the values at the two nodes on either side of the surface (see Fig. 1).

$$\begin{aligned}\phi_e &= \tfrac{1}{2}(\phi_i + \phi_{i+1})\\ \phi_w &= \tfrac{1}{2}(\phi_{i-1} + \phi_i).\end{aligned} \tag{A1}$$

It is known that the direct implementation of the above expression induces instability, see Patankar [11]. To relieve this, Eq. (A1) are rewritten in the same form as Eqs. (2) and (3). That is,

$$\begin{aligned} u>0 \quad &\begin{cases}\phi_e = a_1\phi_{i-1} + a_2\phi_i + a_3\phi_{i+1} + S_e^+\\ \phi_w = b_1\phi_{i-1} + b_2\phi_i + b_3\phi_{i+1} + S_w^+\end{cases}\\ u<0 \quad &\begin{cases}\phi_e = b_3\phi_{i-1} + b_2\phi_i + b_1\phi_{i+1} + S_e^-\\ \phi_w = a_3\phi_{i-1} + a_2\phi_i + a_1\phi_{i+1} + S_w^-,\end{cases}\end{aligned} \tag{A2}$$

where, S_e^+, S_w^+, S_e^-, S_w^- are source terms defined as

$$\begin{aligned} S_e^+ &= -a_1\phi_{i-1} + (1/2 - a_2)\,\phi_i + (1/2 - a_3)\,\phi_{i+1}\\ S_w^+ &= (1/2 - b_1)\,\phi_{i-1} + (1/2 - b_2)\,\phi_i - b_3\phi_{i+1}\\ S_e^- &= -b_3\phi_{i-1} + (1/2 - b_2)\,\phi_i + (1/2 - b_1)\,\phi_{i+1}\\ S_w^- &= (1/2 - a_3)\,\phi_{i-1} + (1/2 - a_2)\,\phi_i - a_1\phi_{i+1}.\end{aligned} \tag{A3}$$

The coefficients a_i and b_i $(i = 1, 2, 3)$ are determined applying the rules (R1)–(R5) as

$$\begin{aligned} a_1 &= 0, \quad a_2 = 1, \quad a_3 = 0 \\ b_1 &= 1, \quad b_2 = 0, \quad b_3 = 0. \end{aligned} \tag{A4}$$

This formulation has been employed as the central difference scheme for convection terms. The resultant solution procedure revealed significantly enhanced stability. It is noted that this central difference formulation of the convection term is essentially that used by Ghia *et al.* [7].

ACKNOWLEDGMENTS

Partial funding for this work was provided by the University Energy Research Group of the University of California and the Institute for Scientific Computing Research at the Lawrence Livermore National Laboratory (University of California). We are grateful to these two agencies for their support and to the CRAY Corporation for the computing time required to perform the study on the Berkeley campus. Many thanks go to Ms. Janet Christian for the preparation of this paper.

REFERENCES

1. B. P. Leonard, *Comput. Methods Appl. Mech. Eng.* **19**, 59 (1979).
2. M. A. Leschziner, *Comput. Methods Appl. Mech. Eng.* **23**, 293 (1980).
3. T. Han, J. A. C. Humphrey, and B. E. Launder, *Comput. Methods Appl. Mech. Eng.* **29**, 81 (1981).
4. A. Pollard and A. L. W. Siu, *Comput. Methods Appl. Mech. Eng.* **35**, 293 (1982).
5. C. J. Freitas, R. L. Street, A. N. Findikakis, and J. R. Koseff, *Int. J. Numer. Methods Fluids* **5**, 561 (1985).
6. C. Y. Perng and R. L. Street, *Int. J. Numer. Methods Fluids* **9**, 341 (1989).
7. U. Ghia, K. N. Ghia, and C. T. Shin, *J. Comput. Phys.* **48**, 387 (1982).
8. R. Schreiber and H. B. Keller, *J. Comput. Phys.* **49**, 310 (1983).
9. Y. Choi, J. A. C. Humphrey, and F. S. Sherman, *J. Comput. Phys.* **75**, 359 (1988).
10. J. Koseff and R. L. Street, *J. Fluids Eng.* **106**, 21 (1984).
11. S. V. Patankar, *Numerical Heat Transfer and Fluid Flow* (Hemisphere, Washington, DC/New York, 1980).
12. M. C. Thompson and J. H. Ferziger, *J. Comput. Phys.* **82**, 94 (1989).
13. G. E. Schneider and M. Zedan, *Numer. Heat Transfer* **4**, 1 (1981).
14. B. P. Leonard, in *Proceedings, 5th Int. Conf. on Numerical Methods in Laminar and Turbulent Flows, 1987*, edited by C. Taylor *et al.* (Pineridge Press, Swansea, UK, 1987), p. 35.